EE 505

Lecture 24

ADC Design – Pipeline

Review from last lecture

Pseudo-Static Time-Invariant Modeling of a Linear Pipelined ADC

Parameterization of Stage k

•Amplifier Closed-Loop Gain •From input – m1k •From DAC – m2k •From offset – m3k •Offset Voltage - V_{OSk} •DAC •V_{DACki} •ADC Offset Voltages - V_{OSAki} •Out-Range Circuit (if used and not included in ADC/DAC) •DAC Levels - V_{DACBki} •Amplifier Gain – m4k



Review from last lecture Solution of the 2n Linear Equations

$$V_{in} = \left\{ \frac{d_{I} \left[\left(\frac{m_{21}}{m_{11}} \right) V_{DAC1} \right] + d_{2} \left[\left(\frac{m_{22}}{m_{11}m_{12}} \right) V_{DAC2} \right] + ... + d_{n} \left[\left(\frac{m_{2n}}{m_{11}m_{12}...m_{1n}} \right) V_{DACn} \right] + \frac{V_{REF}}{2^{n+1}} \right\}$$

$$\left\{ \frac{M_{31}}{m_{11}} V_{OS1} + \frac{m_{32}}{m_{11}m_{12}} V_{OS2} + ... + \left(\frac{m_{3n}}{m_{11}m_{12}...m_{1n}} \right) V_{OSn} \right\}$$

$$\left\{ \frac{V_{RESn}}{m_{11}m_{12}...m_{1n}} - \frac{V_{REF}}{2^{n+1}} \right\}$$

$$Code-independent offset term offset term$$

Note: Will not even include last residue amplifier nor create V_{RESn}

Note: ADC errors do not affect linearity performance of pipelined structure but DAC outputs and weights are critical

Pseudo-Static Time-Invariant Modeling of a Linear Pipelined ADC



If more than 1 bit/stage is used and DAC is binarilyweighted structure

$$V_{_{RESk}} = m_{_{1k}}V_{_{ink}} + m_{_{2k}}\left(\sum_{_{j=1}^{2^{n_{k}}-1}}d_{_{kj}}V_{_{DACkj}}\right) + m_{_{3k}}V_{_{OSk}}$$

Review from last lecture

Pipelined ADC



Claim: If the Amplifiers are linear, settling is complete, and over-range protection is provided, the pipelined ADC makes no nonlinearity errors if the output of the DACs are correctly interpreted

Implication: Flash ADC errors, offsets in comparators and amplifiers, and gain errors in amplifier and S/H do not degrade linearity performance of a well-designed pipelined ADC structure !!

Review from last lecture Observations

$$V_{in} = \sum_{k=1}^{n} \alpha_k d_k + f(\text{offset}) + f(\text{residue})$$

form of $\alpha_k : V_{\text{DACk}} \frac{m_{2k}}{\prod_{j=1}^{k} m_{1j}}$

- Substantial errors are introduced if α_k are not correctly interpreted!
- Some calibration and design strategies focus on accurately setting gains and DAC levels
- Analog calibration can be accomplished with either DAC level or gain calibration
- Digital calibration based upon coefficient identification does not require accurate gains or precise DAC levels

Review from last lecture Observations (cont)

$$V_{in} = \sum_{k=1}^{n} \alpha_k d_k + f(offset) + f(residue)$$

 $\begin{array}{lll} \text{form of} & \alpha_k \, : \, V_{\text{DACk}} \frac{m_{2k}}{\displaystyle\prod_{j=1}^k m_{1j}} \end{array}$

- If nonlinearities are avoided, data conversion process with a pipelined architecture is extremely accurate
- Major challenge at low frequencies is accurately interpreting the digital output codes

Review from last lecture Observations (cont)

$$V_{in} = \sum_{k=1}^{n} \alpha_k d_k + f(offset) + f(residue)$$

 $\begin{array}{lll} \text{form of} & \alpha_k : \ V_{\text{DACk}} \frac{m_{2k}}{\displaystyle\prod_{j=1}^k m_{1j}} \end{array}$

• If nonlinearities are present, this analysis falls apart and the behavior of the ADC is unpredictable !

Intuitive View of Why Sub-ADCs do Not Cause Nonlinearity Errors



Claim: If the Amplifiers are linear, settling is complete, and over-range protection is provided, the pipelined ADC makes no nonlinearity errors if the output of the DACs are correctly interpreted

for 1 bit/stage

$$V_{in} = \left\{ d_{1} \left[\left(\frac{m_{21}}{m_{11}} \right) V_{REF} \right] + d_{2} \left[\left(\frac{m_{22}}{m_{11}m_{12}} \right) V_{REF} \right] + \dots + d_{n} \left[\left(\frac{m_{2n}}{m_{11}m_{12}\dots m_{1n}} \right) V_{REF} \right] + \frac{V_{REF}}{2^{n+1}} \right\} + V_{OSEQ} + \epsilon$$

- ADCs determine whether a quantity is or is not subtracted from V_{REF} at each stage but the DAC determines how much is subtracted
- Keep subtracting smaller-and-smaller quantities from V_{IN} until residue is approx. 0 at end of last stage (and error caused by last sub-ADC will be small)
- If we know how much is subtracted from V_{IN} until residue vanishes, we know V_{IN}
- Over-range protection recovers errors caused by subtracting too much or too little

Performance Limitations of Pipelined ADCs (consider amplifier, ADC and DAC issues)

- ADC - Break Points (offsets) DAC - DAC Levels (offsets) • Out-range (over or under range) Amplifier
 - Offset voltages
 - Settling Time
 - Nonlinearity (primarily open loop)
 - Open-loop
 - Out-range
 - Gain Errors
 - Inadequate open loop gain
 - Component mismatch
 - Power Dissipation
 - kT/C switching noise



Performance Limitations of Pipelined ADCs (consider amplifier, ADC and DAC issues)



- Nonlinearity (primarily open loop)
 - Open-loop
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Pipelined Data Converter Design Guidelines

Issue

Strategy

1. ADC offsets, Amp Offsets, Finite Op Amp Gain, DAC errors, Finite Gain Errors all cause amplifiers to saturate

Typical Finite-Gain Inter-stage Amplifier (shown single-ended with 1-bit/stage)



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But what really happens?

Ideal transfer characteristics (1 bit/stage)

What are the effects of these errors?





Effects of Simultaneous Errors

Ideal transfer characteristics (1 bit/stage)

What are the effects of these errors?









Extra comparator levels in ADC

Ideal transfer characteristics (1 bit/stage)

Over-range Protection





Extra comparator levels in ADC (1 extra comparator)



Extra comparator levels in ADC (2 extra comparators)

Ideal transfer characteristics (1 bit/stage)



Extra comparator levels in ADC (2 extra comparators)





Sub-radix Structure

Pipelined Data Converter Design Guidelines

Issue

1. ADC offsets, Amp Offsets, Finite Op Amp Gain, DAC errors, Finite Gain Errors all cause amplifiers to saturate

Strategy

- 1. Out-range protection circuitry will remove this problem and can make pipeline robust to these effects if α_k 's correctly interpreted
 - a) Use Extra Comparators
 - b) Use sub-radix structures

Pipelined Data Converter Design Guidelines

Issue

- 1. ADC offsets, Amp Offsets, Finite Op Amp Gain, DAC errors, Finite Gain Errors all cause amplifiers to saturate
- 2. Correct interpretation of α_k 's is critical

Strategy

- 1. Out-range protection circuitry will remove this problem and can make pipeline robust to these effects if α_k 's correctly interpreted
 - a) Use Extra Comparators
 - b) Use sub-radix structures
- 2. a) Accurately set α_k values
 - b) Use analog or digital calibration

Performance Limitations of Pipelined ADCs (consider amplifier, ADC and DAC issues)



- Nonlinearity (primarily open loop)
- Open-loop
- ➡ Out-range
- → Gain Errors
 - Inadequate open loop gain
 - Component mismatch
 - Power Dissipation
 - kT/C switching noise

X_{OUTk}

Amplifier Types used In Pipelined ADCs

- Two-stage
- Cascode
 - Telescopic
 - Folded
- Regulated Cascode (Gain-boosted Cascode)
 - Telescopic
 - Folded
- Regenerative Feedback Gain Enhancement
- Two-Stage Cascode



Single-Ended Output



Fully Differential



Fully Differential

Telescopic Cascode



CMFB Not Shown

Folded Cascode Amplifier



CMFB Not Shown

Gain-Boosted Telescopic Cascode



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Gain-Boosted Folded Cascode



-g_m Compensation Implementation



-gm Compensated Single-Stage



Two-Stage Cascode/Cascade



Amplifier Nonlinearity Becoming Increasingly Significant as V_{DD} Reduced Comparison of amplifiers at same power

level and same V_{FB}



Drop in gain seriously degrades linearity and spectral performance

- Nonlinearity strongly architecture dependent
- Trade-Offs between Gain and Signal Swing

How Much Gain?

Depends upon how much of the overall error budget is allocated to the effect noninfinite gain has on required performance parameters

If require n ENOB, can 1/2 LSB be allocated to effects of op amp gain error?

e.g. If INL specification of a 12-bit ADC is ½ LSB, can ½ LSB be allocated to the noninfinite gain error?

Sources that may contribute to INL errors in pipelined ADC:

Finite Op Amp Gain Capacitor Missmatch Incomplete amplifier settling Amplifier nonlinearity Input S/H error Parasitic capacitance nonlinearity Offset voltage (in ADC, DAC, summer) DAC errors ADC nonlinaritry

Error Budgeting

Sources that may contribute to INL errors in pipelined ADC:

Finite Op Amp Gain Capacitor Missmatch Incomplete amplifier settling Amplifier nonlinearity Input S/H error Parasitic capacitance nonlinearity Offset voltage (in ADC, DAC, summer) DAC errors ADC nonlinaritry

If entire error budget (e.g. ½ LSB) is allocated to the Finite Op Amp Gain, what error budget must be allocated to all remaining contributors?

What will happen if each error source is allocated an error budget of (e.g. ½ LSB)?

How should the error sources contribution to overall error budget be allocated?

 $\sum_{i=1}^{m} e_i = \frac{1}{2}LSB \quad \text{(maybe a little bit overly conservative)}$

How Much Gain?

Conventional Approach: Assume want to make at most ½ LSB error in closed loop gain at each stage

Often see authors use
$$A_{dB} \cong 6n_{sT} + 12$$

- Gives no information about drop in gain at boundary of input/output window
- Maybe uses too much error budget on gain
- Errors accumulate since gain errors will exist on each stage
- No indication how A_{dB} relates to INL or DNL
- Gain requirements are large on the input buffer ($n_{ST}=n$) but will be significantly relaxed on latter stages in the pipeline when n_{ST} decreases

Pipelined Data Converter Design Guidelines

Issue

- 1. ADC offsets, Amp Offsets, Finite Op Amp Gain, DAC errors, Finite Gain Errors all cause amplifiers to saturate
- 2. Op Amp Gain causes finite gain errors and introduces noninearity

Strategy

- 1. Out-range protection circuitry will remove this problem and can make pipeline robust to these effects if α_k 's correctly interpreted
 - a) Use Extra Comparators
 - b) Use sub-radix structures
- 2. a) Select op amp architecture that has acceptable signal swing

b) Select gain large enough at boundary of range to minimize nonlinearity and gain errors

Performance Limitations of Pipelined ADCs (consider amplifier, ADC and DAC issues)



- Nonlinearity (primarily open loop)
- ➡ Open-loop
- → Out-range
- → Gain Errors

- Inadequate open loop gain
- Component mismatch
- Power Dissipation
- kT/C switching noise

X_{OUTk}



- Can show that no distortion is introduced in pipelined ADC if the amplifier settling is linear (i.e. don't worry about incomplete settling)
- But invariably slew rate and op amp nonlinearities will cause settling to be nonlinear
- Since can't guarantee linear settling, must design for complete settling

Amplifier Settling Time

Worst Case Settling

- Neglect over-range protection (could be up against over-range limit)
- Occurs when input causes <u>output</u> to swing from 0 to V_{REF}





Amp

X_{INk}

CLK

ADC,

d_k

XOUTK

Conventional Approach: Assume want to make at most ½ LSB error in settling for worst-case step on output in each stage

Note: This may not be quite good enough since allocating total error budget to settling of each stage



Compensated Operational Amplifier can be approximately modeled by

$$\mathsf{A}_{OL}(\mathsf{s}) \cong \frac{\mathsf{A}_{\mathsf{o}}\mathsf{p}_{\mathsf{F}}}{\mathsf{s+p}_{\mathsf{F}}} = \frac{\mathsf{G}\mathsf{B}}{\mathsf{s+p}_{\mathsf{F}}}$$

Conventional Approach: Assume want to make at most ½ LSB error in settling for worst-case step in each stage



$$A_{OL}(s) \cong \frac{A_{O}p_{OL}}{s+p_{OL}} = \frac{GB}{s+p_{OL}} \qquad A_{FB}(s) = \frac{A_{O}p_{OL}}{s+p_{OL}+\beta A_{O}p_{OL}} \cong \frac{GB}{s+\beta GB}$$

Step response (if slewing is neglected and dc gain large)

$$r(t)=F+(I-F)e^{-\beta GBt_{s}}$$

$$V_{REF}(1-\varepsilon)=V_{REF}(1-e^{-\beta GBt_{s}})$$

$$1-\varepsilon=1-e^{-\beta GBt_{s}}$$

$$\varepsilon=e^{-\beta GBt_{s}}$$

 $t_{s} = -\frac{\ln(\varepsilon)}{\beta GB}$

or, in terms of the time constant τ of closed loop amplifier

$$t_s = -\tau \ln(\varepsilon)$$
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Conventional Approach: Assume want to make at most ½ LSB error in settling for worst-case step in each stage

Define n_{ST} to be the number of bits of resolution at the residue output of a stage



$$\mathbf{t}_{s} \cong 0.7 (n_{s\tau} + 1) \tau$$

- linear increase in settling requirements with n_{ST}
- n_{ST} determined by accuracy requirements at residue output of a stage

Still need design requirements for GB of Op Amp

$$\mathsf{t}_{s} \cong \frac{0.7(\mathsf{n}_{s\tau}+1)}{\beta \mathsf{GB}}$$

Conventional Approach: Assume want to make at most ½ LSB error in settling for worst-case step in each stage



Note: GB requirements drop from stage to stage

Conventional Approach: Assume want to make at most ½ LSB error in settling for worst-case step in each stage



Compensated Operational Amplifier can be approximately modeled by

$$A(s) \cong \frac{A_{o}p_{F}}{s+p_{F}} = \frac{GB}{s+p_{F}}$$

What about high-impedance op amp?

Conventional Approach: Assume want to make at most ½ LSB error in settling for worst-case step at each stage



What about high-impedance op amp driving capacitive load (including β network)?



Conventional Approach: Assume want to make at most ½ LSB error in settling for worst-case step



Note this is identical in form to that from the internally compensated op amp ⁵³

Conventional Approach: Assume want to make at most ½ LSB error in settling for worst-case step



Notes: May be over-using error budget Slewing will modestly slow response

Pipelined Data Converter Design Guidelines

Issue

1. ADC offsets, Amp Offsets, Finite Op Amp Gain, DAC errors, Finite Gain Errors all cause amplifiers to saturate

2. Op Amp Gain causes finite gain errors and introduces noninearity

3. Op amp settling must can cause errors

Strategy

- 1. Out-range protection circuitry will remove this problem and can make pipeline robust to these effects if α_k 's correctly interpreted
 - a) Use Extra Comparators
 - b) Use sub-radix structures
- 2. a) Select op amp architecture that has acceptable signal swing

b) Select gain large enough at boundary of range to minimize nonlinearity and gain errors

3. Select GB to meet settling requirements (degrade modestly to account for slewing)

Performance Limitations of Pipelined ADCs (consider amplifier, ADC and DAC issues)



- Nonlinearity (primarily open loop)
 - ➡ Open-loop
 - → Out-range
- → Gain Errors

- Inadequate open loop gain
- Component mismatch
- Power Dissipation
 - kT/C switching noise

X_{OUTk}



Dominant source of power dissipation is in the op amps in S/H and individual stages

- Power dissipation strongly dependent upon op amp architecture and design
- Power budgets critical and even a net 5% savings in power is significant!

Consider a single stage in the pipeline



Consider single stage open-loop op amp structures (e.g. telescopic cascode)

$$A_{OL} = -\frac{g_{MT}}{g_{OT}} \qquad P_{OP AMP} \cong 2I_{DQ} \left(\bigvee_{DD} - \bigvee_{SS} \right)$$
Power increases linearly with I_{DQ}
For MOS implementation with basic reference SE op amp
$$= -\frac{2I_{DQ}}{V_{EB}} \frac{1}{2\lambda I_{DQ}} = -\frac{1}{\lambda V_{EB}}$$
No power implications on dc gain of op amp
$$= -\frac{2I_{DQ}}{V_{EB}} \frac{1}{2\lambda I_{DQ}} = -\frac{1}{\lambda V_{EB}}$$
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For MOS implementation (with ref SE op amp or telescopic cascode op amp)

$$GB = \frac{2I_{DQ}}{V_{EB}C_{L}} = \left(\frac{1}{(V_{DD} - V_{SS})}\frac{P}{C_{L}}\right)\frac{1}{V_{EB}}$$
$$P = V_{SUP} \bullet GB \bullet C_{L} \bullet V_{EB}$$

For convenience, define

$$V_{SUP} = V_{DD} - V_{SS}$$

- P increases linearly with GB
- Keep V_{EB} small, C_L as small as possible, GB as small as possible
- At high speeds, diffusion parasitics will cause P to increase more rapidly than GB
- Total amplifier power is sum of power in each stage



For single-stage MOS implementation (with ref SE op amp or telescopic cascade op amp)



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For Single-stage MOS implementation (with ref SE op amp or telescopic cascade op amp)



architecture-dependent term



For Single-stage MOS implementation (with ref SE op amp or telescopic cascade op

$$P = \begin{bmatrix} V_{SUP} \bullet GB \bullet C_{L} \end{bmatrix} \begin{bmatrix} V_{EB} \end{bmatrix} \qquad GB_{HZ} \cong \frac{0.22(n_{ST} + 1)}{\beta} f_{CLK}$$

Fixed by ADC requirements

- n_{ST} =n for S/H thus S/H is a major power consumer
- Use energy efficient op amp architecture
- Power increases linearly with GB (even faster at high frequencies)
- Interleaving can reduce power dissipation at high frequencies(and extend effective clock speed)
- Power increases linearly with clock speed (or worse at high frequencies)
- Power can be scaled down in latter stages since ${\rm n}_{\rm ST}$ will decrease
- Amplifiers can be shared between stages or switched off when not used (factor of 2!)
- Using more than one bit/stage will reduce power since no of op amps will decrease (offsets decrease in β)
- Elimination of S/H will have dramatic effect on power reduction



For Single-stage MOS implementation (with ref SE op amp or telescopic cascade op amp)

$$\mathsf{P} = \begin{bmatrix} \mathsf{V}_{\mathsf{SUP}} \bullet \mathsf{GB} \bullet \mathsf{C}_{\mathsf{L}} \end{bmatrix} \begin{bmatrix} \mathsf{V}_{\mathsf{EB}} \end{bmatrix} \qquad \mathsf{GB}_{\mathsf{HZ}} \cong \frac{\mathsf{0.22}(\mathsf{n}_{\mathsf{sT}} + 1)}{\beta} \mathsf{f}_{\mathsf{CLK}}$$

Fixed by ADC requirements

Which op amp architectures are most energy efficient?

- Depends upon β
- For smaller β , two-stage are more energy efficient for larger β single-stage are better
- Must optimize power in any given architecture
- Folding reduces efficiency (typically by 30% to 50%)



Stay Safe and Stay Healthy !

End of Lecture 24

End of Lecture 21 From Spring 2019